

$$\dot{\epsilon} = \frac{1}{\Omega} \frac{dY}{dt} = \frac{1}{\Omega} \frac{dY}{d\ln t} = \frac{1}{\Omega} \frac{dY}{d\ln a} = \frac{1}{\Omega} \frac{dY}{d\ln R} = \frac{1}{\Omega} \frac{dY}{d\ln R_H} = \frac{1}{\Omega} \frac{dY}{d\ln R_{\text{hor}}}$$